Seismic Refraction I

ESS 314 Geophysics · University of Washington

Week 2, Lecture 6 · April 9, 2026

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By the end of this lecture…

- **[LO-6.1]** *Sketch* a refraction survey geometry and identify direct, reflected, and head wave ray paths
- **[LO-6.2]** *Derive* travel-time equations for direct and head waves in a two-layer model
- **[LO-6.3]** *Distinguish* the critical distance x_{crit} from the crossover distance x_{cross}
- **[LO-6.4]** *Calculate* t_i, x_{crit}, and x_{cross}; invert the T(x) plot for V_1, V_2, and H
- **[LO-6.5]** *Identify* assumptions of the two-layer model and when they fail

The discovery: Mohorovičić, 1909

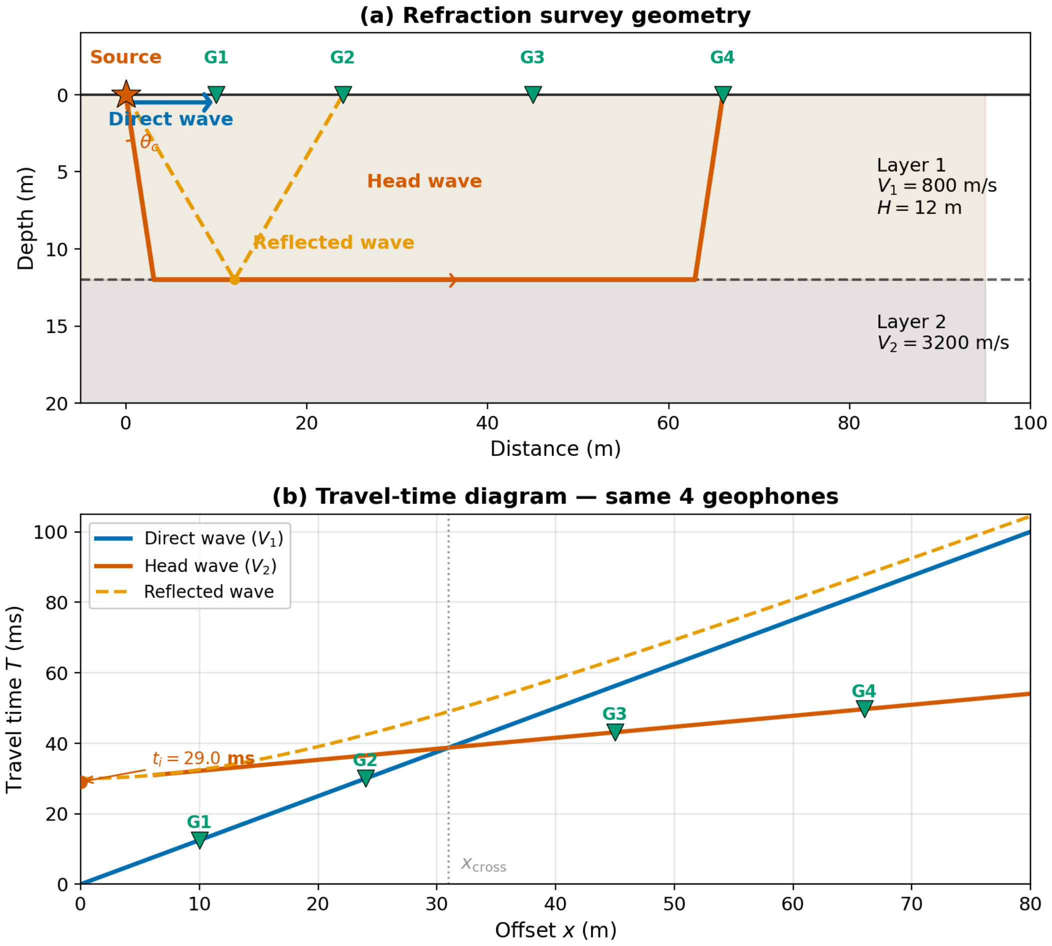
A Croatian geophysicist examines seismograms from an earthquake near Zagreb.

- **Nearby stations:** P-wave slope $\approx 1/5.6$ km/s — crustal rock
- **Distant stations:** a faster P-wave appears — slope $\approx 1/8.1$ km/s
- Beyond ~200 km, the fast wave arrives **before** the direct wave

From slopes and intercept → **crust-mantle boundary ≈ 54 km depth**

This is the **Mohorovičić discontinuity** — discovered by seismic refraction.

The refraction survey



Three types of arrivals

Arrival	Path	Speed	T(x) shape
Direct	Along surface in layer 1	V_1	Linear, origin
Reflected	Down to interface, back up	V_1	Hyperbola
Head wave	Down at θ_c, interface at V_2, up at θ_c	V_2	Linear, intercept

The refraction method exploits **first arrivals** — the earliest energy at each geophone.

Equipment

Source: Sledgehammer on plate (~30 m) · weight drop (~100 m) · explosives (km-scale)

Receivers: 12–48 geophones at 2–5 m spacing, or DAS fiber-optic cable

Recording: Multichannel seismograph — digitizes all channels simultaneously

At crustal scale: explosions or earthquakes as sources; 100s of stations over 100s of km profiles.

Direct wave travel time

The direct wave travels horizontally through layer 1 at velocity V_1:

$$T_{\text{direct}}(x) = \frac{x}{V_1}$$

A straight line through the origin with **slope** $= 1/V_1$.

The near-offset branch of the T(x) plot gives V_1 directly from its slope.

Head wave: three-segment ray path

1. **Down** through layer 1 at θ_c: horizontal reach $= H\tan\theta_c$
2. **Along** the interface: distance $= x - 2H\tan\theta_c$, speed V_2
3. **Up** through layer 1 at θ_c: symmetric to segment 1

$$T_{\text{head}} = \frac{2H}{V_1\cos\theta_c} + \frac{x - 2H\tan\theta_c}{V_2}$$

Apply $\sin\theta_c = V_1/V_2$ and $1 - \sin^2\theta_c = \cos^2\theta_c$:

$$\boxed{T_{\text{head}}(x) = \frac{x}{V_2} + \underbrace{\frac{2H\cos\theta_c}{V_1}}_{t_i}}$$

⚠️

$\cos\theta_c$ is in the **numerator**.

Intercept time and layer depth

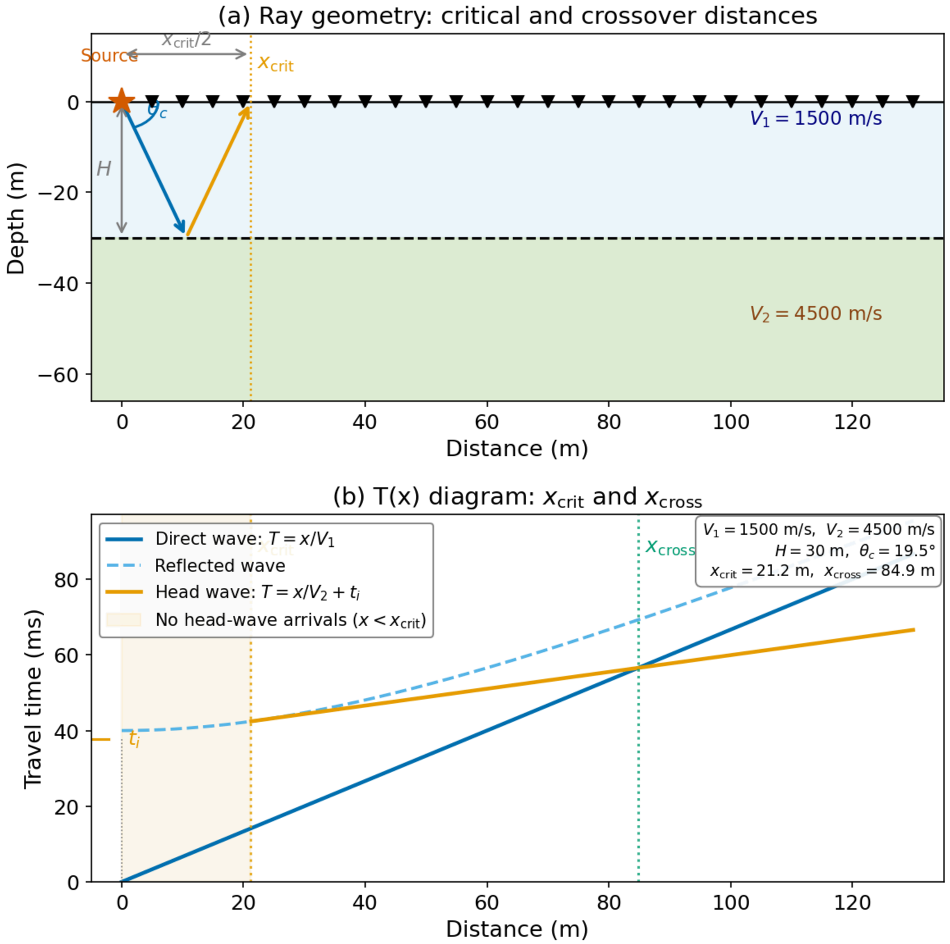
$$t_i = \frac{2H\cos\theta_c}{V_1} = \frac{2H\sqrt{V_2^2 - V_1^2}}{V_1 V_2}$$

Solving for H:

$$\boxed{H = \frac{t_i\, V_1\, V_2}{2\sqrt{V_2^2 - V_1^2}}}$$

Once slopes give V_1 and V_2, the intercept time t_i completely determines the layer thickness H.

Critical distance and crossover distance



x_{crit} vs. x_{cross}

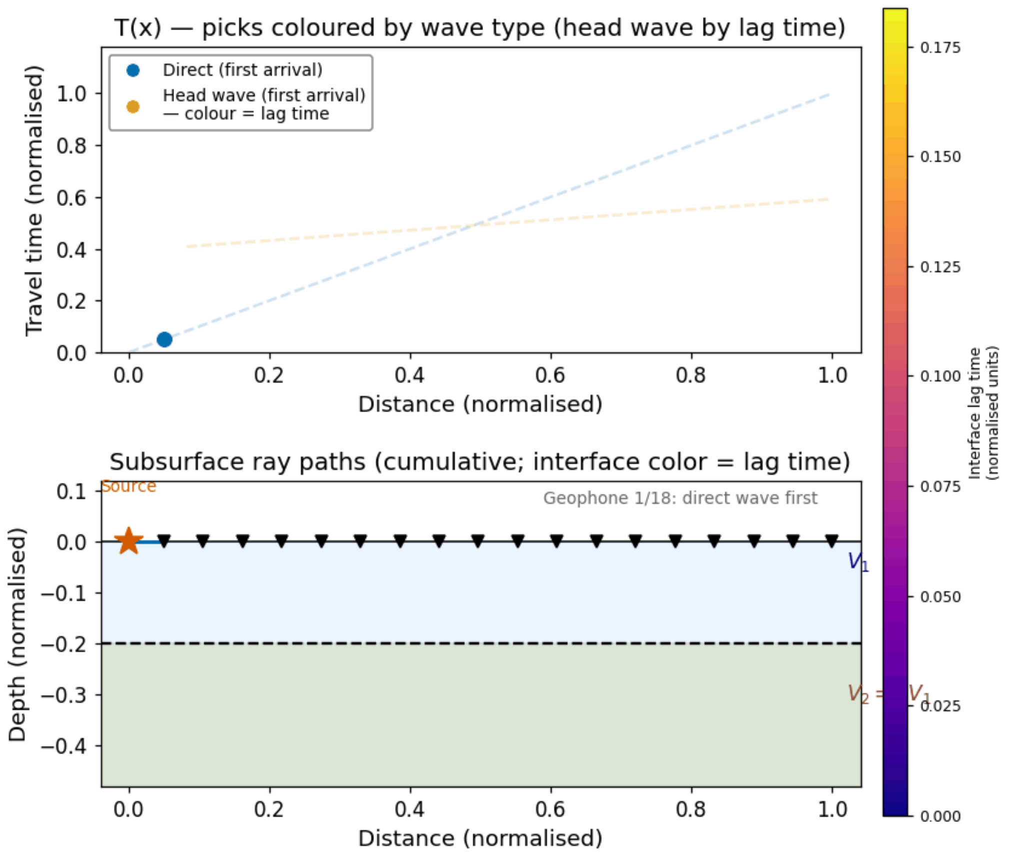
Quantity	Physical meaning	Formula
$x_{\text{crit}} = 2H \tan \theta_c$	Min. offset: head wave **exists**	geometry
$x_{\text{cross}} = 2H\sqrt{(V_2 + V_1)/(V_2 - V_1)}$	Offset: head wave is **first arrival**	$T_d = T_h$

Always: $x_{\text{crit}} < x_{\text{cross}}$

Between these two offsets the head wave exists but arrives **after** the direct wave.

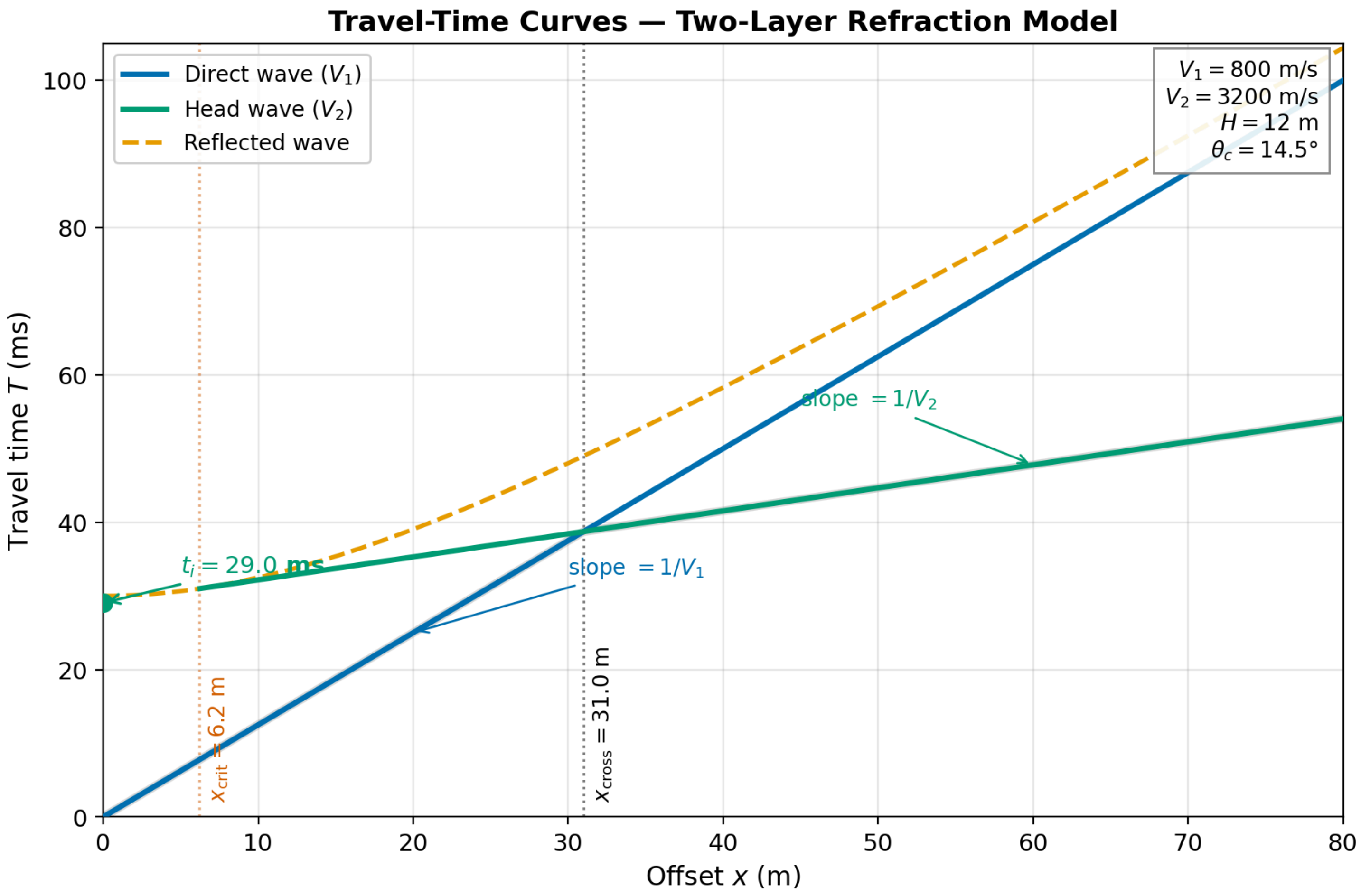
Survey design: the geophone array must reach at least x_{cross} or the inversion fails.

How the T(x) diagram is built



The **plasma color** = interface lag time. Same scale in both panels.

The T(x) plot: all three branches

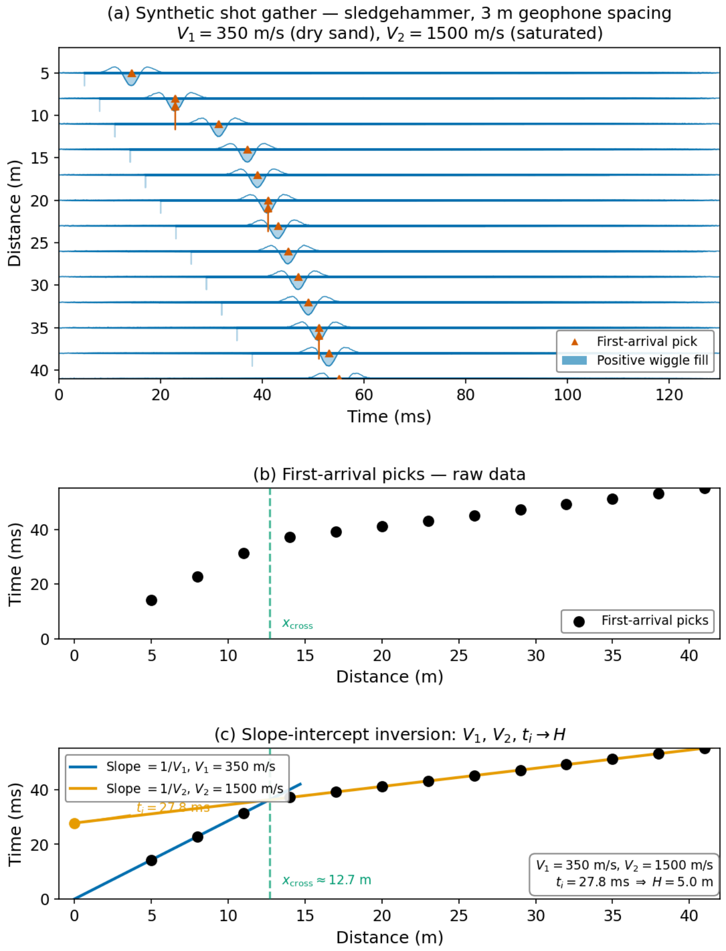


The slope-intercept inversion

From first arrivals to Earth model:

1. Pick first arrivals on the shot gather
2. Plot T vs. x and identify the two linear branches
3. Near-offset slope $\Rightarrow V_1$; far-offset slope $\Rightarrow V_2$
4. Read t_i from the T-axis intercept of the head-wave line
5. Compute $H = t_i V_1 V_2 / (2\sqrt{V_2^2 - V_1^2})$

Synthetic shot gather and inversion



Field inversion: water table at 5 m

Near-offset slope $\Rightarrow V_1 = 350$ m/s (dry sand)

Far-offset slope $\Rightarrow V_2 = 1500$ m/s (saturated sand)

Intercept: $t_i = 27.8$ ms, $\theta_c = \arcsin(350/1500) = 13.5^\circ$

$$H = \frac{0.0278 \times 350 \times 1500}{2\sqrt{1500^2 - 350^2}} = 5.0 \text{ m}$$

The velocity jump arises because water's bulk modulus dominates the pore space — the V_P physics from Lecture 4 applied at field scale.

Worked example: UW campus bedrock

$V_1 = 800$ m/s (glacial till) · $V_2 = 3200$ m/s (bedrock) · $H = 12$ m

$$\theta_c = 14.5^\circ \qquad t_i = 29.0\ \text{ms} \qquad x_{\text{crit}} = 6.2\ \text{m} \qquad x_{\text{cross}} = 31.0\ \text{m}$$

Offset	T_{direct}	T_{head}	First arrival
$x = 30$ m	37.5 ms	38.4 ms	direct
$x = 36$ m	45.0 ms	40.3 ms	**head wave**

The crossover at 31 m falls between these two receivers — consistent with the prediction.

Assumptions and failure modes

Assumption	Consequence if violated
$V_2 > V_1$	No critical refraction → no head wave → layer invisible
Flat interface	Dipping layers distort apparent slopes; depth estimate is biased
Homogeneous layers	Velocity gradients curve rays; T(x) is non-linear
First arrivals only	Later arrivals carry additional structure; reflection profiling needed

Hidden layer: a thin low-velocity unit between two faster layers generates no first-arrival head wave and is entirely invisible to refraction. Addressed in Lecture 7.

Societal relevance: PNW applications

Depth to bedrock: Sound Transit used refraction to map bedrock along Seattle light rail alignments — directly informing cut-and-cover vs. bored-tunnel decisions.

Water table: The 350 → 1500 m/s jump is one of the strongest refraction signals in near-surface geophysics; used routinely for contamination plume monitoring.

Crustal thickness: Moho depth beneath the Pacific Northwest — ~10 km under the ocean, ~40 km under the Cascades — is mapped from earthquake refraction arrivals recorded by the PNSN.

AI as a reasoning partner

Prompt to evaluate:

> *"Derive the head-wave travel time for a two-layer model. Show the three path segments, simplify using $\sin\theta_c = V_1/V_2$, and derive x_{crit}."*

Criteria for a correct response:

- $\cos\theta_c$ appears in the **numerator** of t_i
- $x_{\text{crit}} = 2H\tan\theta_c$ is derived geometrically, not confused with x_{cross}
- The step $1 - \sin^2\theta_c = \cos^2\theta_c$ is shown explicitly

Concept Check

1. $V_1 = 2000$ m/s, $V_2 = 5500$ m/s, $H = 50$ m. Calculate θ_c, t_i, x_{crit}, x_{cross}.
2. In three sentences: why does the head wave — which travels a longer total path — arrive first at distant receivers?
3. Slopes 5.0 ms/m and 1.25 ms/m, $t_i = 30$ ms. Determine V_1, V_2, H.
4. A survey records no head-wave arrivals. List three physically distinct explanations.

Next time

Lecture 7 — Seismic Refraction II

What happens when the interface is **dipping**? When there are **multiple layers**? When a layer is **hidden**?

Forward and reverse shooting · dipping-layer geometry · the plus-minus method · hidden layers and velocity inversions

Lab 2 (Friday): Python II — implementing the travel-time equations and forward ray tracing