

Seismic Refraction II

Beyond the Flat Layer: Special Cases and Uncertainty

ESS 314 Geophysics · University of Washington

Week 3, Lecture 7 · April 13, 2026

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By the end of this lecture...

- [LO-7.1] *Derive* the N -layer travel-time generalization and the dipping-interface equations
- [LO-7.2] *Explain* why low-velocity zones and thin layers are invisible to refraction surveys
- [LO-7.3] *Apply* the delay-time method to map irregular refractors from reversed profiles
- [LO-7.4] *Enumerate* principal sources of data uncertainty and their effect on depth estimates
- [LO-7.5] *Implement* a forward model predicting T - x curves for layered and dipping geometries

Where We Left Off

Single-layer horizontal model (Lecture 6):

$$t_2(x) = \frac{x}{V_2} + \frac{2h_1 \cos \theta_{ic}}{V_1}, \quad \theta_{ic} = \sin^{-1} \frac{V_1}{V_2}$$

- Slope \Rightarrow velocity; intercept \Rightarrow depth
- Works when: horizontal layers, monotonically increasing velocity

Today: What happens when these assumptions fail?

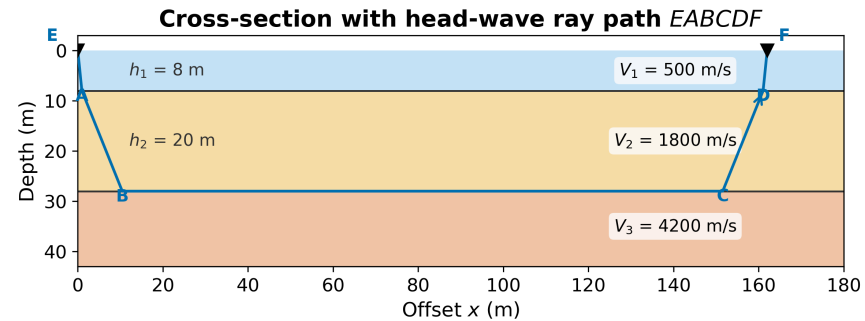
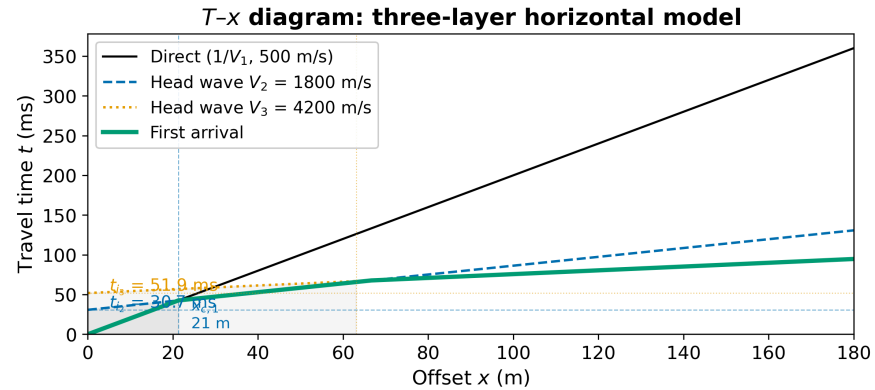
Multi-Layer Generalization

For N horizontal layers ($V_1 < V_2 < \dots < V_N$):

$$t_n(x) = \frac{x}{V_n} + \frac{2}{V_n} \sum_{i=1}^{n-1} h_i \frac{\sqrt{V_n^2 - V_i^2}}{V_i}$$

- Each head wave yields one T - x segment with slope $1/V_n$
- Intercept times solved **sequentially**: h_1 from t_{i_2} , then h_2 from t_{i_3} using known h_1 , etc.
- Layer thicknesses are **not** independent: every deeper estimate depends on all shallower ones

Multi-Layer $T-x$ Diagram



Three layers, three slope segments. The first head wave from each interface is the first arrival only beyond its crossover distance.

[Python-generated: `assets/scripts/fig_multilayer_traveltime.py`]

Complication 1: Low-Velocity Zone

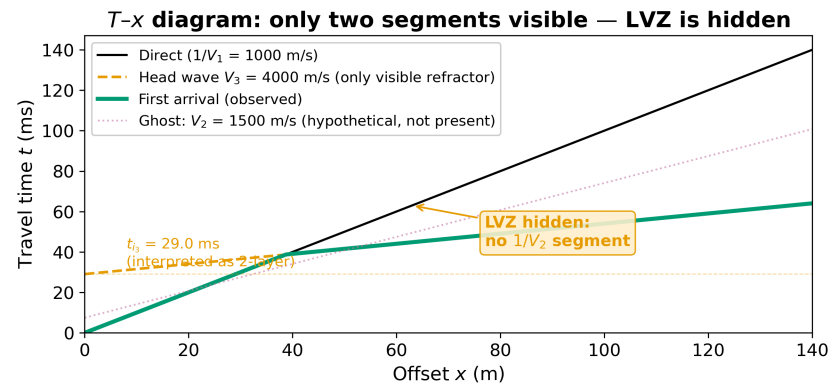
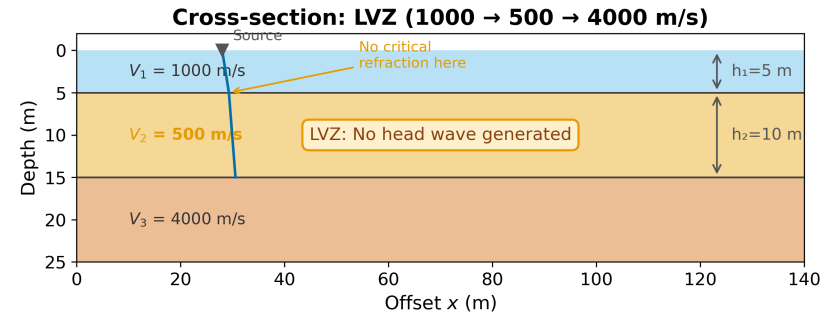
What if $V_2 < V_1$?

- $\sin \theta_{ic} = V_1/V_2 > 1 \rightarrow$ **no critical angle exists**
- No head wave from the V_1 – V_2 interface
- The intermediate layer is **invisible** to refraction

Consequence: The T - x diagram looks like a simple two-layer Earth. The interpreted depth to V_3 is too large. There is **no warning** in the data.

Common cause: saturated clays over indurated bedrock; gas-bearing sands; weathered zones

LVZ: The Hidden Layer



No $1/V_2$ segment appears. P-wave first-arrival refraction alone cannot detect the LVZ.

[Python-generated: `assets/scripts/fig_lvz_traveltime.py`]

Detecting the LVZ: What Works?

P-wave first-arrival refraction **cannot** detect an LVZ — but these methods can:

Method	Why it works
Seismic reflection	Needs only impedance contrast $Z = \rho V$, not $V_2 > V_1$
Refraction tomography (SRT)	Inverts all first arrivals for smooth velocity model
MASW (surface waves)	Rayleigh dispersion is independent of the head-wave condition
S-wave refraction	Only if $V_{S,2} > V_{S,1}$ while $V_{P,2} < V_{P,1}$ — not a general fix

Reflection is the direct remedy. **MASW** is the most powerful for velocity inversions.

Complication 2: Thin Intermediate Layer

Even when $V_1 < V_2 < V_3$, a thin layer may be undetectable.

The V_2 head wave is first arrival only over a **limited offset window**:

$$x_{c,1} = 2h_1 \sqrt{\frac{V_2 + V_1}{V_2 - V_1}}$$

Rule of thumb: Layer n is detectable only if the window width $\gtrsim \Delta x_{station}$.

For typical ratios: station spacing $\lesssim 0.6 h_n$.

Complication 3: Dipping Interface

Dipping layers still produce head waves — but apparent velocity **depends on shooting direction**.

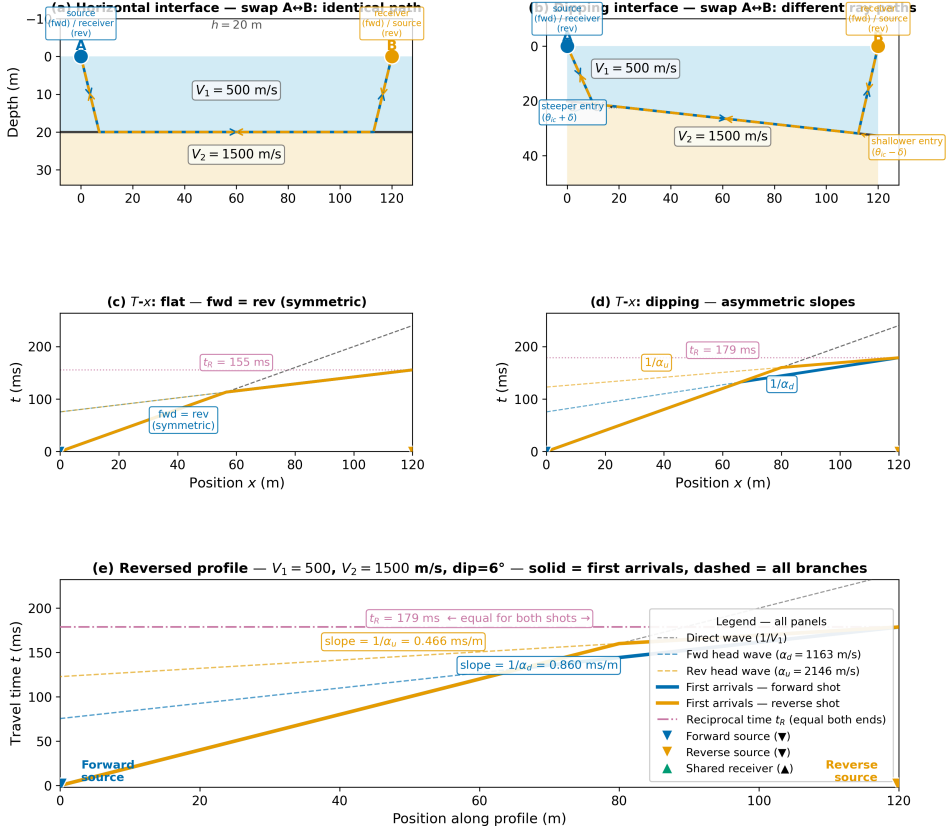
Down-dip: $t_d(x) = \frac{x}{V_1} \sin(\theta_{ic} + \delta) + t_{id}$

Up-dip: $t_u(x) = \frac{x}{V_1} \sin(\theta_{ic} - \delta) + t_{iu}$

- Down-dip: $\alpha_d < V_2$ (underestimates true refractor velocity)
- Up-dip: $\alpha_u > V_2$ (overestimates true refractor velocity)

Solution: shoot from both ends (reversed profile)

Dipping Interface: Reversed Profile



Reversed profiling resolves the ambiguity between dip and velocity.

[Python-generated: `assets/scripts/fig_dipping_interface_reversed.py`]

Recovering True Velocity and Dip

From apparent velocities α_d (down-dip) and α_u (up-dip):

$$\delta = \frac{1}{2} \left[\sin^{-1} \frac{V_1}{\alpha_d} - \sin^{-1} \frac{V_1}{\alpha_u} \right]$$

For small dips ($\delta \lesssim 15\text{--}20^\circ$):

$$\frac{1}{V_2} \approx \frac{1}{2} \left(\frac{1}{\alpha_d} + \frac{1}{\alpha_u} \right)$$

Reciprocal time check: Travel time from source A to far receiver must equal travel time from source B to near receiver. Failure indicates timing error or lateral velocity variation.

The Delay-Time Method

For irregular refractors, the **delay time** at geophone G is:

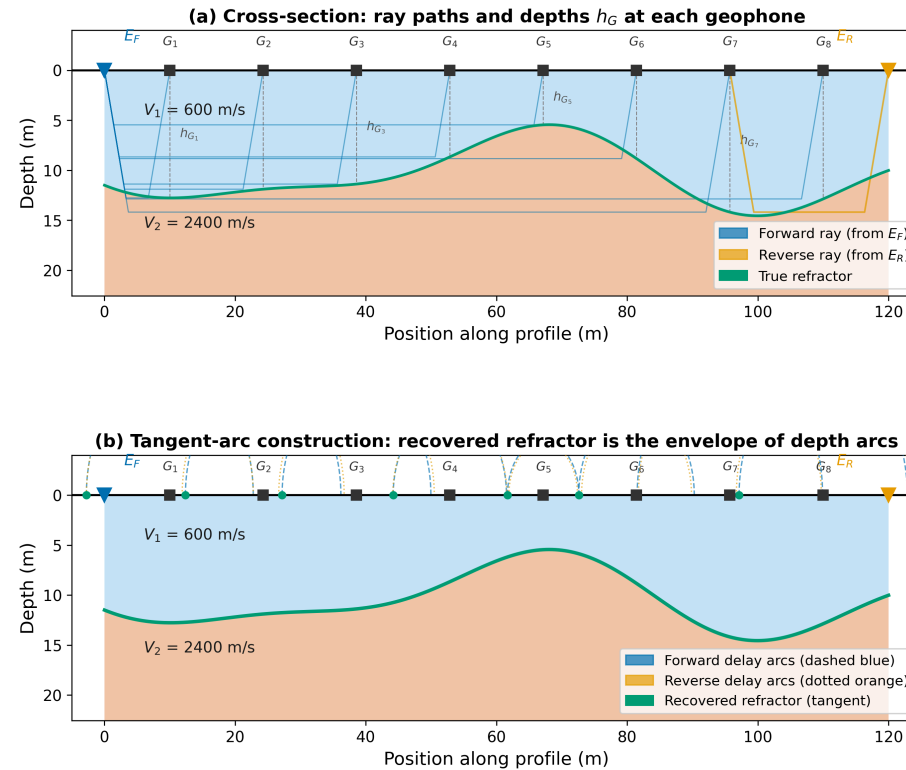
$$\delta t_G = \frac{h_G \cos \theta_{ic}}{V_1}$$

Depth from **both** forward and reverse delay times:

$$h_G = \frac{V_1 V_2}{2\sqrt{V_2^2 - V_1^2}} [\delta t_{F,G} + \delta t_{R,G}]$$

Result: a point-by-point refractor profile beneath every geophone position.

Delay-Time Method: The Tangent-Arc Construction



The refractor surface is the common tangent to all depth arcs.

[Python-generated: `assets/scripts/fig_delay_time_method.py`]

Sources of Uncertainty

Source	Effect	Magnitude
First-arrival picking error (± 1 ms)	Depth error $\delta h = V_1 \delta t / 2 \cos \theta_{ic}$	0.1–5 m
Velocity gradient in top layer	Curved direct-wave segment; biased intercepts	Depends on gradient
Lateral velocity variation	Apparent dip artifact; false structure	Can be large
LVZ (undetected)	Systematic underestimate of depth to refractor	Proportional to LVZ thickness
Station spacing too large	Missing intermediate layer	$\delta h \sim \Delta x / 0.6$



Non-uniqueness: different model combinations can fit the same T-x data within noise — always integrate with borehole and independent geophysical data.

Worked Example: Puget Lowland

Layer	Geology	Velocity
1	Loose fill / organic soil	350 m/s
2	Dense glacial outwash gravel	1650 m/s
3	Renton Fm. sandstone	4200 m/s

From field T - x slopes and intercepts: $h_1 = 1.1$ m, $h_2 = 14.6$ m

Bedrock at ~15.7 m depth — but is there a LVZ in the gravel? Is the bedrock dipping toward the Seattle Fault?

→ Reversed profiling and borehole control needed for reliable site characterization.

Applications in the Pacific Northwest

- **Liquefaction hazard:** Depth to water table and bearing material in Holocene sediments beneath Seattle
- **Debris flow characterization:** Bedrock-colluvium interface on Cascades volcano flanks
- **Transportation:** WSDOT/Sound Transit tunnel and light-rail corridor characterization
- **Seattle Basin:** Sediment-bedrock interface controls site amplification for Cascadia megathrust events

Concept Check

1. A T - x diagram shows only two linear segments even though a borehole shows three velocity units. Name two geological explanations and describe how you would distinguish them.
2. A reversed refraction profile gives apparent velocities $\alpha_d = 1163$ m/s and $\alpha_u = 2146$ m/s with overburden $V_1 = 500$ m/s. Calculate the refractor dip angle δ and true velocity V_2 .
3. A geophone array has 3 m spacing. A thin gravel layer with $V_2/V_1 = 2$ is suspected. What is the minimum thickness you could confidently detect?