Seismic Reflections I

Flat-Layer Travel Time, NMO, and CMP Stacking

ESS 314 Geophysics · University of Washington

Week 3, Lecture 8 · April 15, 2026

Marine Denolle

By the end of this lecture…

[LO-8.1] *Derive* the normal-incidence reflection coefficient from boundary conditions; compute energy reflection coefficient

[LO-8.2] *Derive* the flat-layer hyperbola $t^2 = t_0^2 + x^2/V_1^2$ from the image-point construction

[LO-8.3] *Apply* NMO correction to a CMP gather; explain why stacking improves SNR by $\sqrt{N_\mathrm{fold}}$

[LO-8.4] *State* the RMS velocity definition; apply Dix equation to recover interval velocities

[LO-8.5] *Interpret* a semblance panel to pick stacking velocities

Before the Equations — Look at This

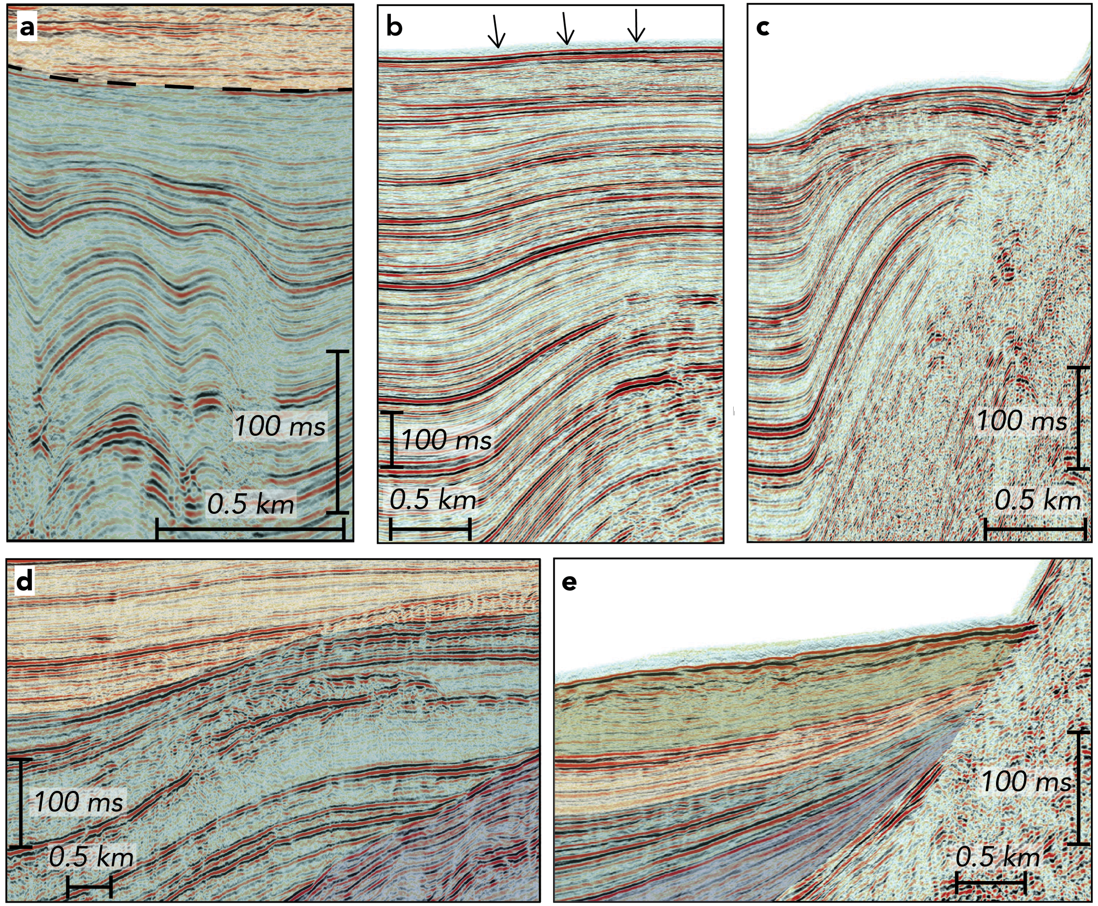


Fig. 2 from Ledeczi et al. (2024), *Seismica* 2(4), doi:10.26443/seismica.v2i4.1158 · Licensed CC BY 4.0 · Reproduced unmodified

Reading a Seismic Section

Orientation

- **Horizontal axis:** distance along the survey line (km)
- **Vertical axis:** two-way travel time (TWTT) in seconds — time increases **downward**; shallower reflectors plot at top
- Each column of pixels = one **seismic trace** recorded at one location

Amplitude and polarity

- **Bright reflector** → large impedance contrast between layers
- **Dark zone** → gradational boundary, low contrast, or gradual velocity change
- **Polarity** (light vs. dark wiggle) → sign of $R = (Z_2 - Z_1)/(Z_2 + Z_1)$:
  - positive R if impedance increases with depth
  - negative R if impedance decreases

> Note: TWTT is *not* depth — converting it to depth requires knowing the P-wave velocity field (exactly what this lecture develops)

From a Single Point to the Cross-Section

Step 1 — Single interface, normal incidence:

A downgoing pulse hits a boundary → fraction R of the energy reflects upward → arrives at the surface at two-way time $t_0 = 2h/V$.

One reflection = one amplitude value at one (x, t_0) pixel.

Step 2 — One vertical trace:

Stack all reflections from all interfaces at location x → a time-series (one column of the image).

Step 3 — Sweep across the survey line:

Repeat for every source–receiver pair → mosaic all traces side by side → the **2D seismic section**.

What controls brightness?

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

High-contrast boundary (e.g., sand–shale, water–rock) → bright.

Small contrast (clay–clay) → dark.

What Makes a Bright Reflector? Acoustic Impedance

The brightness in the Cascadia image is controlled by **acoustic impedance** $Z = \rho V_P$ — the resistance of a medium to wave propagation.

At normal incidence (boundary conditions: continuity of pressure + particle velocity):

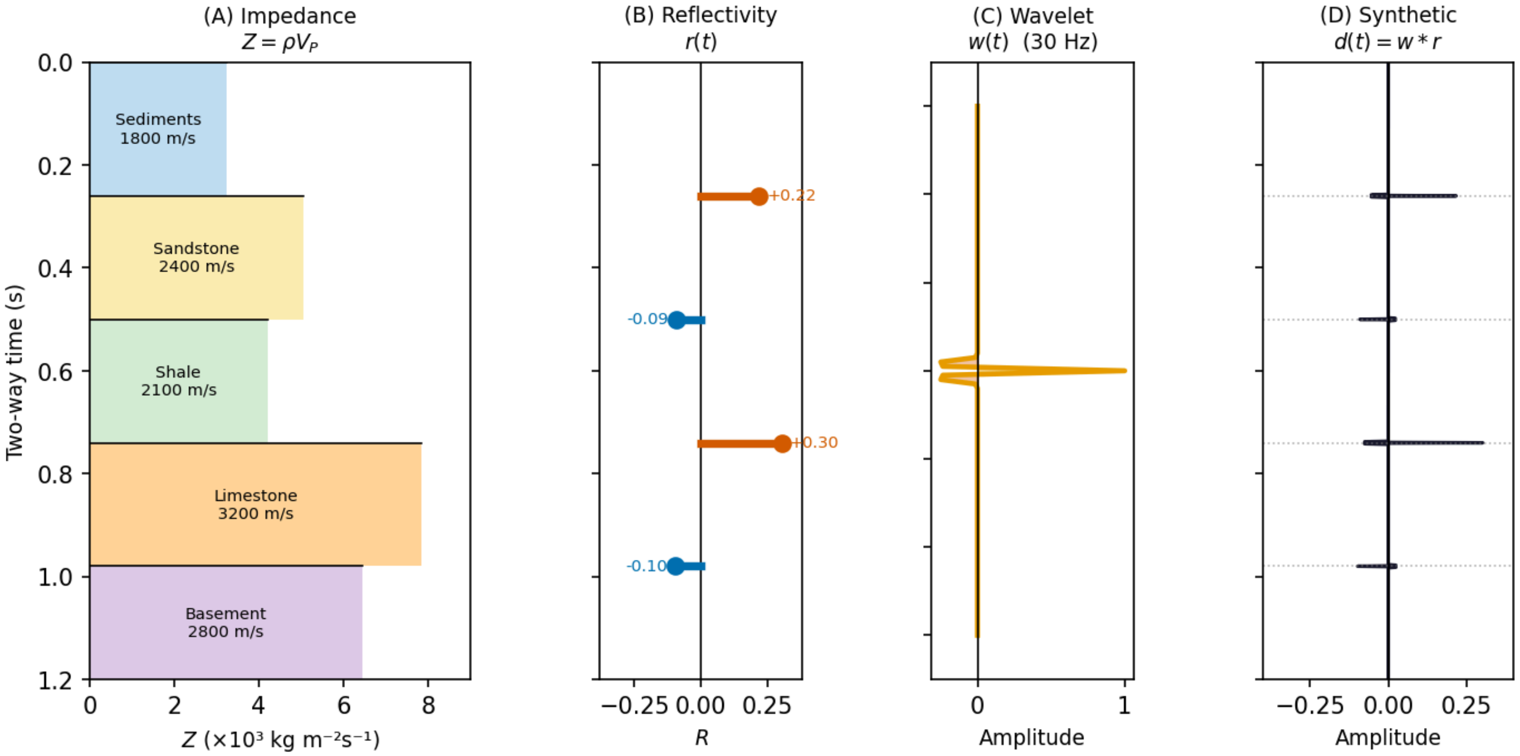
$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad T = \frac{2Z_2}{Z_1 + Z_2}$$

Energy fractions: $\mathcal{R} = R^2, \mathcal{T} = 1 - R^2$

Interface	Z_1 (MPa·s/m)	Z_2	R
Sediment → limestone	3.5	6.0	+0.26
Sand → shale	4.8	4.2	−0.07
Crust → mantle (Moho)	13	20	+0.21

Most sedimentary contacts: $|R| = 0.01$–0.15 → only 1–2% of energy reflected. **CMP stacking is essential to extract the signal.**

From Impedance to Seismogram: The Convolutional Model



$d(t) = w(t) * r(t)$ → each reflector prints a copy of the wavelet, scaled by R and shifted to its TWTT. Blue = positive R (impedance increase); red = negative R.

Why Stack? The Signal-to-Noise Problem

Most sedimentary reflectors have $|R| = 0.01\text{–}0.15$: only **1–2% of seismic energy** reflects at any one boundary — the rest propagates deeper or scatters as noise.

A single source–receiver trace is almost never enough to detect a reflector above ambient noise.

Solution: record the same reflection from N_{fold} different source–receiver offsets. Signal adds **coherently** ($\propto N$); noise adds **incoherently** ($\propto \sqrt{N}$):

$$\text{SNR}_{\text{stack}} = \sqrt{N_{\text{fold}}} \times \text{SNR}_{\text{single}}$$

48-fold → **7×**

96-fold → **10×**

240-fold → **15×**

"Aligning" traces from different offsets to the same reflection time requires understanding the geometry of offset travel time — the **reflection hyperbola**.

The Reflection Hyperbola

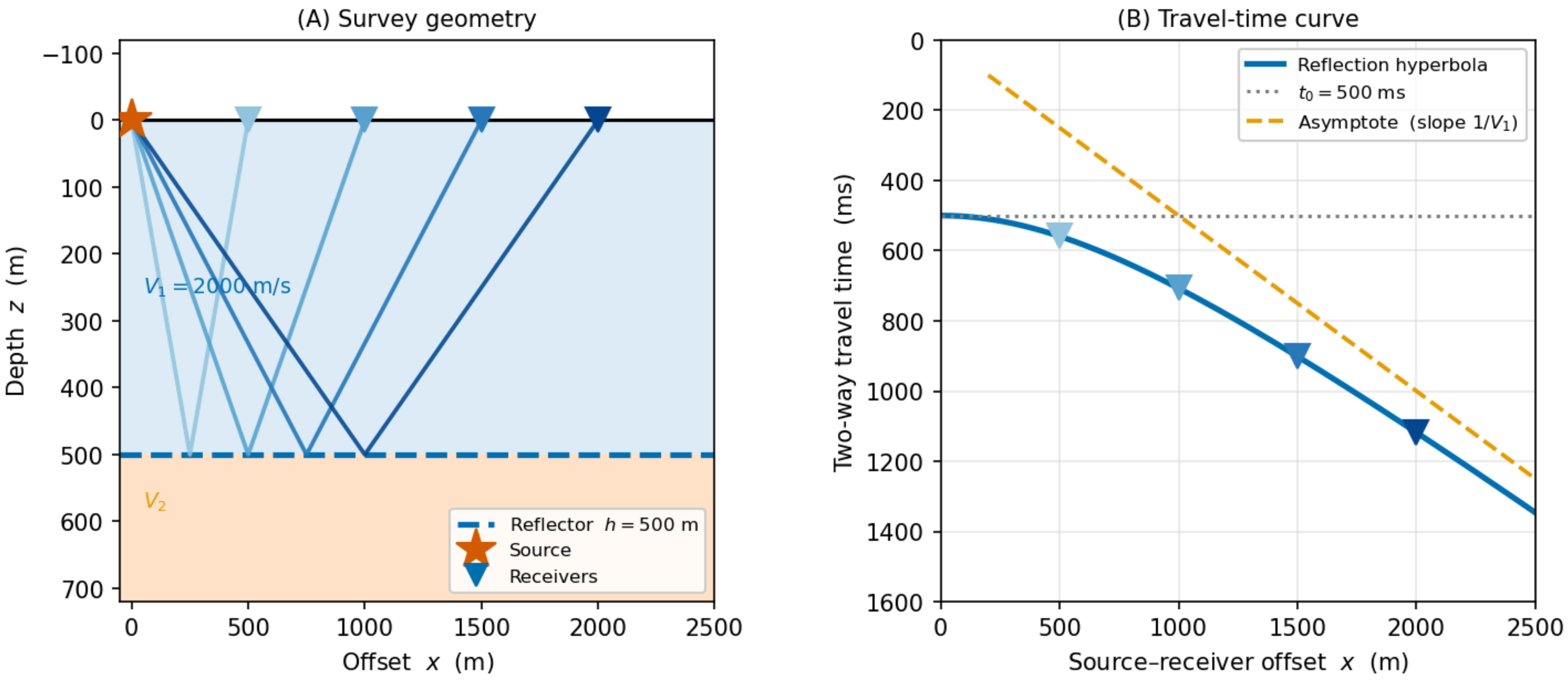


Image-point construction: reflect source through reflector.

$$t^2(x) = t_0^2 + \frac{x^2}{V_1^2}$$

$$t_0 = \frac{2h}{V_1}$$

NMO Correction

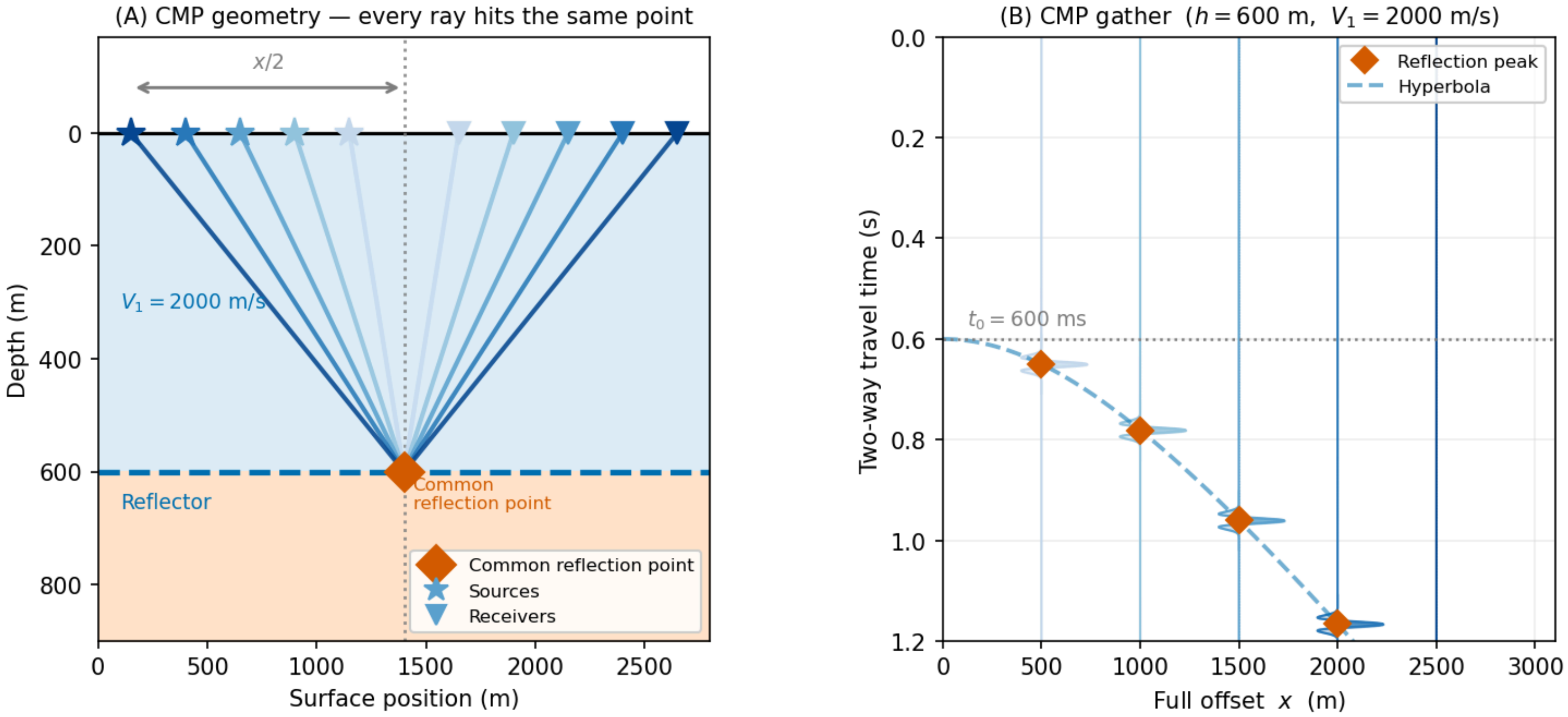
Normal moveout is the delay at offset x relative to t_0:

$$\Delta t_{\mathrm{NMO}}(x) = \sqrt{t_0^2 + \frac{x^2}{V_{\mathrm{NMO}}^2}} - t_0 \approx \frac{x^2}{2\,V_{\mathrm{NMO}}^2\,t_0}$$

NMO correction **shifts each trace up** by $\Delta t_{\mathrm{NMO}}(x)$, flattening the hyperbola to t_0 — this is the alignment step that makes stacking work.

NMO stretch at large offsets distorts the wavelet. Traces beyond the mute zone ($x/h \gtrsim 1\text{–}1.5$) are discarded before stacking.

Acquisition: CMP Gather



Every coloured pair shares the same **common midpoint** (CMP) → same reflection point on a flat reflector. NMO-correcting and summing these traces produces one stacked trace. Fold = spread / (2 × shot spacing). Modern marine: 120–240-fold.

RMS Velocity

For N flat, horizontal layers with velocities V_i and two-way times Δt_i:

$$V^2_{\mathrm{rms,n}} = \frac{\displaystyle\sum_{i=1}^{n} V_i^2 \, \Delta t_i}{\displaystyle\sum_{i=1}^{n} \Delta t_i}$$

- V_{rms} **replaces** V_1 in the hyperbola for multi-layer media
- $V_{\mathrm{rms}} \geq$ any interval velocity above the interface (RMS > arithmetic mean for increasing-velocity profiles)
- The NMO velocity measured from semblance = V_{rms}

The Dix Equation

Recover **interval velocity** between two adjacent reflectors:

$$V_n = \sqrt{\frac{V_{\text{rms,n}}^2\, t_{0,n} - V_{\text{rms,n}-1}^2\, t_{0,n-1}}{t_{0,n} - t_{0,n-1}}}$$

Key assumptions: flat, horizontal, isotropic layers — violations are addressed in Lecture 9.

Precision matters: a small error in V_{rms} propagates strongly to V_n for thin layers (when $t_{0,n} - t_{0,n-1}$ is small).

Velocity Analysis: Semblance Panel

Semblance $S(V,\tau)$: coherence of the NMO-corrected CMP gather at trial velocity V and time τ.

$$S(V,\tau) = \frac{\left[\sum_j d_j(\tau + \Delta t_j)\right]^2}{N \sum_j \left[d_j(\tau + \Delta t_j)\right]^2} \in [0,1]$$

Reading the semblance panel:

- **Pick maxima** tracing a velocity function $V(t_0)$ from shallow to deep
- Velocity should **increase with depth** for a normal gradient profile
- **Multiples** appear at lower velocity than primaries at the same t_0

CMP Stack: The Full Pipeline

After semblance velocity picking, NMO correction, and mute:

$$s(t) = \frac{1}{N_{\text{fold}}} \sum_{j=1}^{N_{\text{fold}}} d_j^{\text{NMO}}(t)$$

Each CMP produces one stacked trace. Assembled side by side → the **2D stacked section** (the cross-section we opened the lecture with).

Post-stack: deconvolution → **migration** (Lecture 10) → interpretation

Worked Example: Two-Layer NMO + Dix

$V_1 = 1800$ m/s, $h_1 = 900$ m; $V_2 = 2600$ m/s, $h_2 = 700$ m

Reflector	t_0 (s)	V_{rms} (m/s)
1	1.000	1800
2	1.538	2048

Dix recovery: $V_2 = \sqrt{(2048^2 \times 1.538 - 1800^2 \times 1.000)/0.538} = 2600$ m/s ✓

SOTA: DL Velocity Analysis

Traditional velocity picking: manual, time-consuming, subjective for millions of CMPs in 3D surveys.

CNN semblance pickers: input = semblance image; output = $V(t_0)$ curve. Match expert picks within 1–2% RMS.

Bayesian uncertainty: output $p(V_{\mathrm{NMO}} \mid t_0)$ — widest uncertainty at large TWTTs and near-zero fold zones.

Physics-constrained inversion: embed Dix equation as a hard constraint → interval velocities guaranteed consistent with observed V_{rms}.

Concept Check

1. Two layers: $V_1 = 2000$ m/s at $t_0 = 0.80$ s; $V_{\mathrm{rms},2} = 2300$ m/s at $t_0 = 1.40$ s. Compute V_2 with Dix. Compute the depth to reflector 2.
2. NMO is applied to a CMP gather using a velocity that is 5% too low. Are the hyperbolas over- or under-corrected? What does the gather look like after correction?
3. A semblance panel shows a peak at $(V = 2000\ \mathrm{m/s},\ t_0 = 1.0\ \mathrm{s})$ and another at $(V = 1500\ \mathrm{m/s},\ t_0 = 2.0\ \mathrm{s})$. What is the second event most likely to be?
4. A 48-fold stack has $\mathrm{SNR}_{\mathrm{single}} = 0.5$. What is $\mathrm{SNR}_{\mathrm{stack}}$? Is the reflector visible?