Building Earth Images

The Iterative Refraction–Reflection Workflow

ESS 314 — Introduction to Geophysics

Lecture 10 · Monday 4/27/2026

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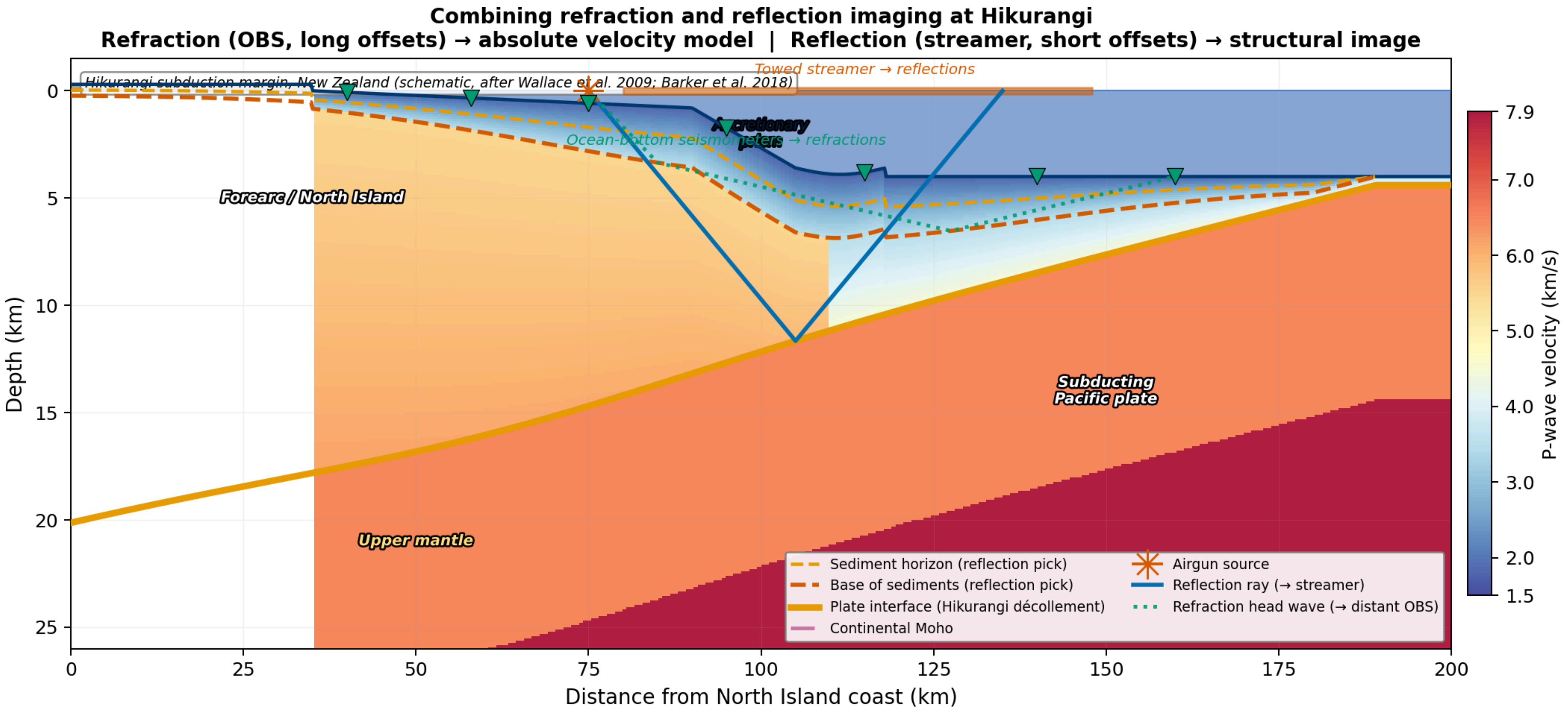
Learning objectives

By the end of this lecture:

- Explain the difference between **forward modeling** ($\mathbf{d} = F\mathbf{m}$) and **migration** ($\hat{\mathbf{m}} = F^\top \mathbf{d}$)
- Show why **even flat reflectors** need a layered velocity model to be correctly depth-converted
- Derive the migration corrections $\Delta x = d \sin\theta$ and $\tau = t \cos\theta$ for dipping layers
- Describe how Kirchhoff migration **collapses diffractions** to their source points
- **Diagnose** from a migrated image whether the velocity was correct, too slow (frowns), or too fast (smiles)
- Apply the **8-step iterative workflow** combining refraction and reflection
- Evaluate a deep-learning surrogate: who provided its training data?

The Hikurangi subduction margin



Pacific plate subducts beneath North Island, New Zealand.

How do we image the plate interface at 5–20 km depth?

Same airgun shots → refractions to OBS (long offset) + reflections to streamer (short offset)

Two windows, one Earth

Refraction imaging (OBS, long offsets)

→ absolute layer velocities $v_1, v_2, \ldots$

→ reliable to ~1–10 km depth

Reflection imaging (streamer, shorter offsets)

→ structural image at all depths

→ velocities are relative, not absolute

Neither method alone gives a quantitatively correct depth image.

Their combination, iterated, does.

The framework: forward and inverse modeling

	Forward model	Inverse / adjoint
Question	Given $\mathbf{m}$, predict $\mathbf{d}$?	Given $\mathbf{d}$, estimate $\mathbf{m}$?
Operator	F (physics → data)	$F^\top$ (data → image)
Example	$t = 2z/v$, ray tracing	Kirchhoff sum, RTM
Requires	Model + velocity v	Data + velocity v
Output	Synthetic seismogram	Depth image

$$\mathbf{d} = F\,\mathbf{m} \qquad \hat{\mathbf{m}} = F^\top\,\mathbf{d}$$

Both need $v(x, z)$. Migration ($F^\top$) is not the true inverse — it's the adjoint. Its quality depends entirely on the velocity model.

Building the image: four cases

Case	Complication	Key equation	Migration needed?
1	Flat layer, constant v	$z = vt/2$	Trivial (time → depth)
2	Multiple flat layers	Dix + refraction	Yes — need layered v
3	Dipping layers	$\Delta x = d \sin\theta, \tau = t \cos\theta$	Yes — mispositioning
4	Diffractions	Kirchhoff sum along hyperbola	Yes — collapse to point

Each case adds one physical complication. Each case shows the same lesson: **you need an accurate velocity model.**

Case 1 — Flat layer, constant velocity

For a flat reflector at depth z, velocity v, zero-offset geometry:

$$t = \frac{2z}{v} \quad \Longrightarrow \quad z = \frac{vt}{2}$$

- Normal ray is **vertical** — the display is correct
- "Migration" = multiply by $v/2$ → **exact time-to-depth conversion**
- No horizontal shift, no depth error

This is why intro courses skip migration for flat layers. The display is already right.

Case 2 — Multiple flat layers: the velocity matters

Three layers with velocities $v_1 < v_2 < v_3$, interfaces at $z_1 < z_2 < z_3$:

$$t_1 = \frac{2z_1}{v_1}, \quad t_2 = t_1 + \frac{2(z_2 - z_1)}{v_2}, \quad t_3 = t_2 + \frac{2(z_3 - z_2)}{v_3}$$

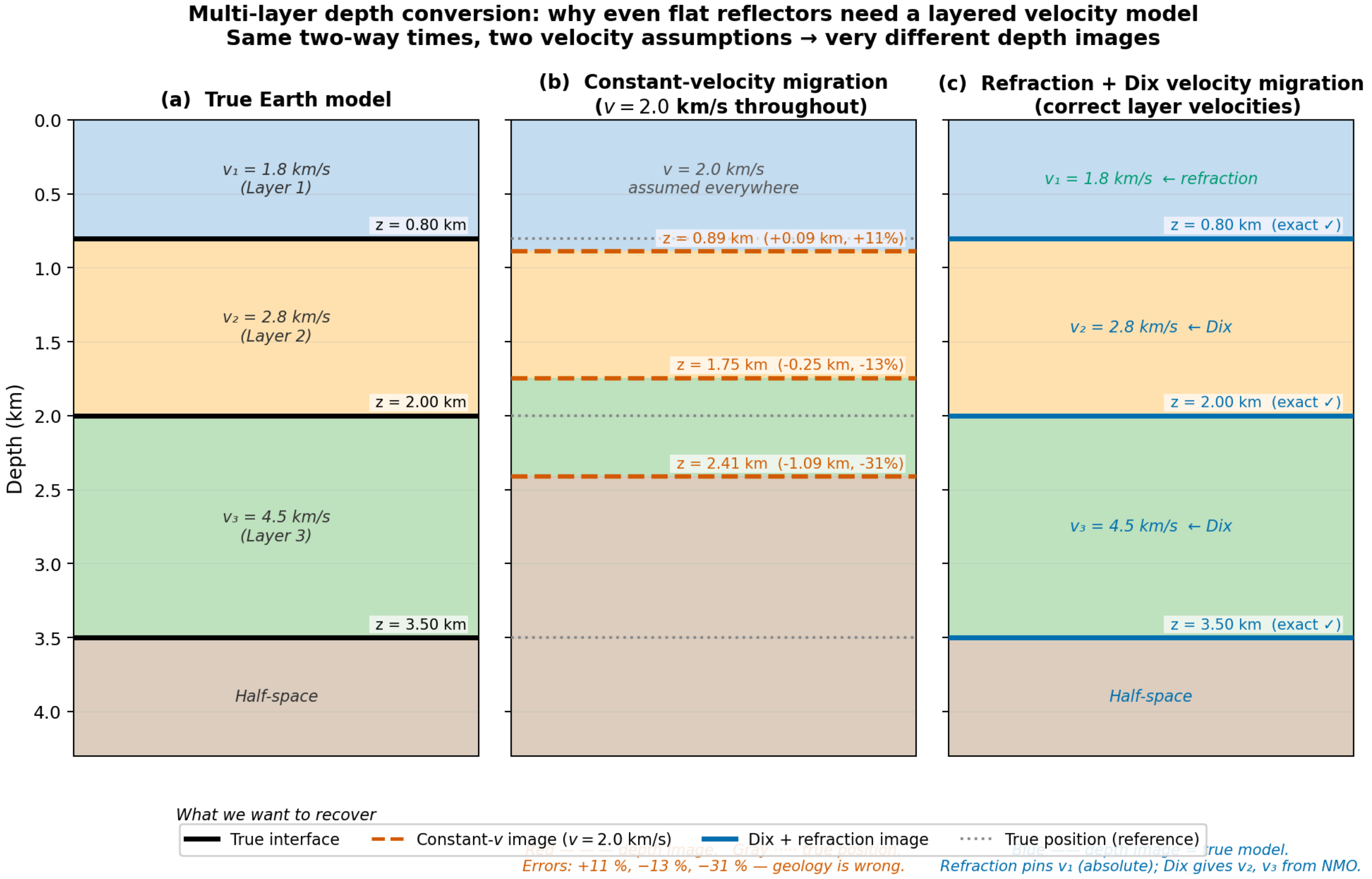
To recover depths → need interval velocities v_1, v_2, v_3 via Dix:

$$v_n^2 = \frac{V_{\mathrm{rms},n}^2\, t_n - V_{\mathrm{rms},n-1}^2\, t_{n-1}}{t_n - t_{n-1}}$$

Problem: Dix integrates downward. Error in v_1 propagates into v_2, v_3.

Solution: Refraction gives absolute v_1 → anchors the chain.

Case 2 — The depth image depends on velocity



Same two-way times. Two velocity assumptions. Very different images.

Constant $v = 2.0$ km/s → deepest interface 31% wrong.

Refraction + Dix → all three interfaces exact.

Case 3 — Dipping layers: mispositioning

For a dipping reflector (θ from horizontal), the normal ray is not vertical.
The instrument records only $t = 2d/v$ — no directional information.
Conventional display: plot event straight down at depth d. **Two errors result:**

$$\Delta x = d \sin\theta \quad \text{(too far downdip)}$$

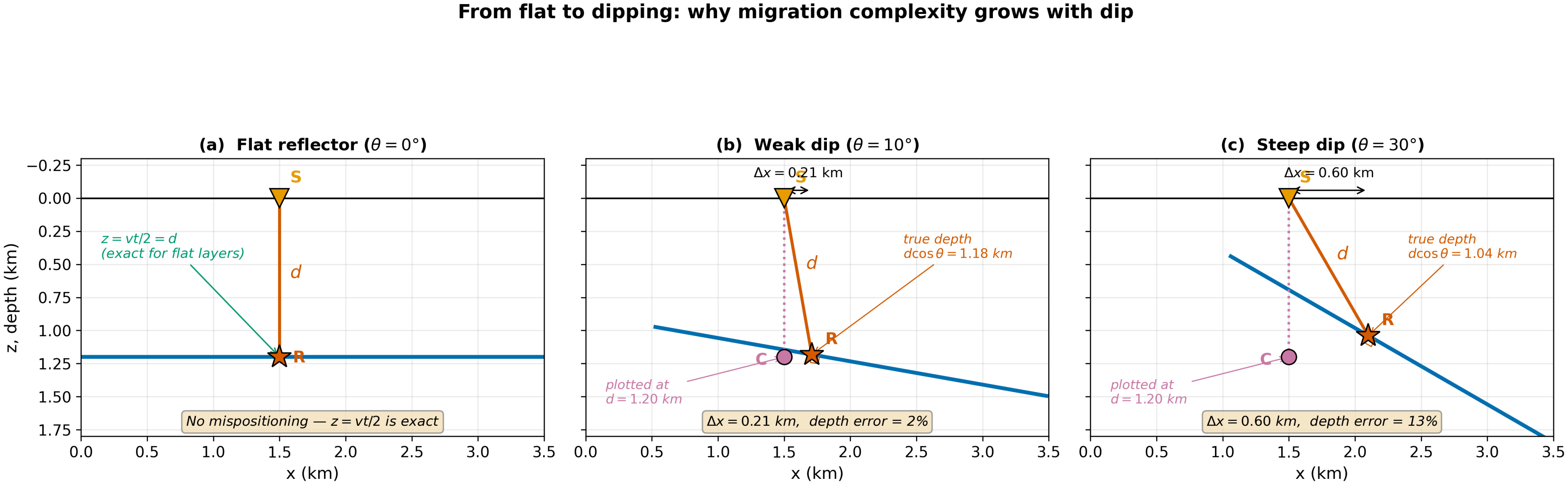
$$\tau = t\cos\theta \quad \text{(corrected time, shallower depth)}$$

Hand-migration formulas (using slope $p_0 = \partial t / \partial y$):

$$\Delta x = \frac{v^2 p_0 t}{4}, \qquad \tau = t\sqrt{1 - \frac{v^2 p_0^2}{4}}$$

Both $\to 0$ when $\theta \to 0$ (flat-layer limit).

Case 3 — Flat to dipping: error grows with dip



Dip θ	Δx	Depth error
0°	0	0%
10°	0.21 km	2%
30°	0.60 km	13%

Migration applies the corrections. Both corrections depend on **the velocity** — again.

Case 4 — Diffractions: signature of structure

Any **geometric discontinuity** (fault tip, unconformity edge, salt flank) generates a diffraction.

In zero-offset data, a point scatterer at (x_0, z_0) produces a **hyperbola**:

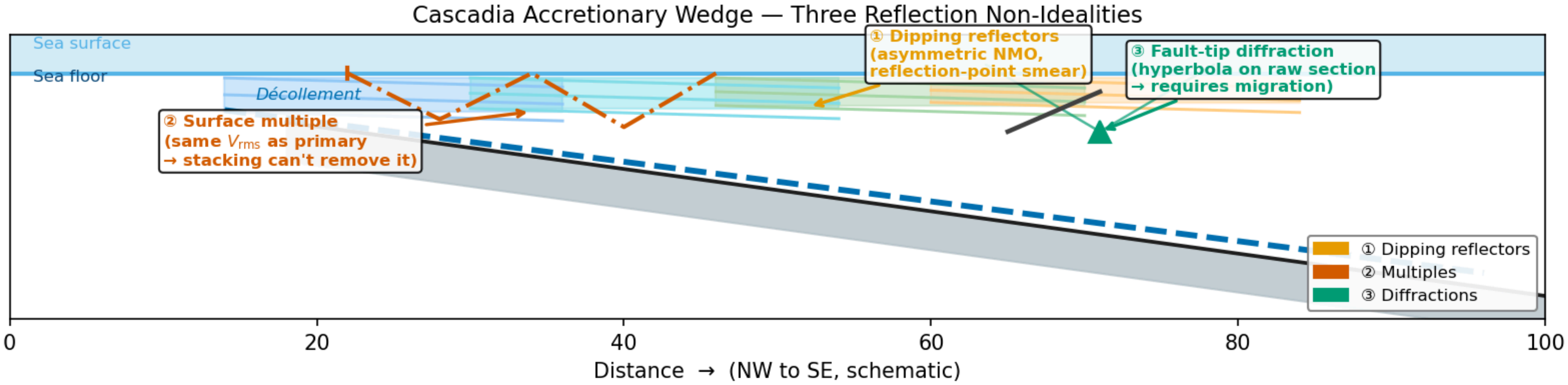
$$t(y) = \sqrt{\left(\frac{2z_0}{v}\right)^2 + \left(\frac{2(y - x_0)}{v}\right)^2}$$

An unprocessed section over complex geology is full of overlapping hyperbolas.

Three challenges in a real accretionary wedge:

- Dipping reflectors → mispositioning (Case 3)
- Surface multiples → same V_{rms}, hard to remove
- **Diffraction hyperbolas → geological structure hidden until migrated**

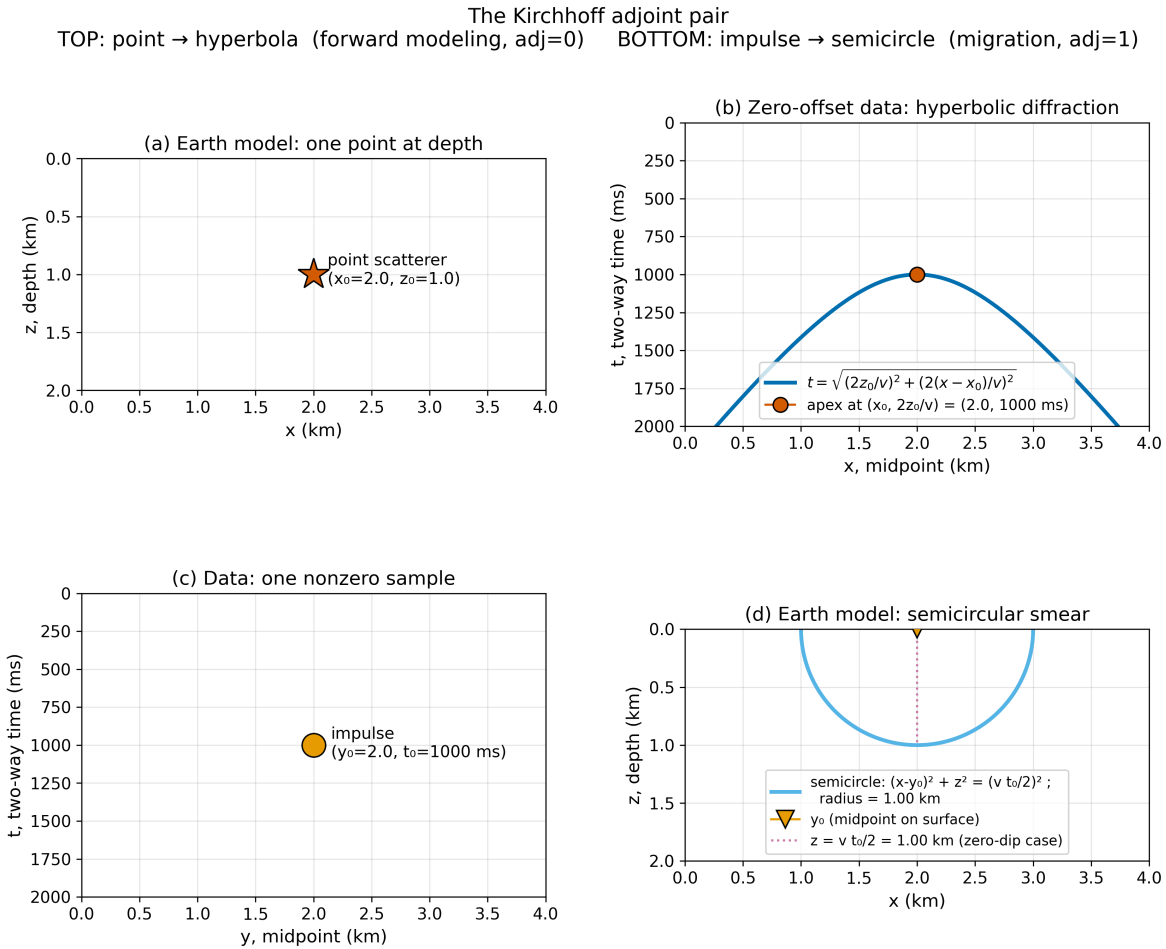
Case 4 — Diffractions in the Cascadia wedge



The diffraction hyperbola at the fault tip is not noise — it **is the fault**.

Kirchhoff migration collapses it to a point at the fault tip location.

Kirchhoff migration: the adjoint pair



Forward F: scatterer $(x_0, z_0) \to$ hyperbola in data. Writes energy along the curve.

Migration $F^\top$: sums data along hyperbola $\to$ focused point. Reads energy along the same curve.

Kirchhoff in pseudocode

```
for every (ix, iz) in the model:
    for every midpoint y in the data:
        t = sqrt( (2·z[iz]/v)² + (2·(x[ix]−y)/v)² )   # same hyperbola!
        if forward:   data[t, y]    += model[iz, ix]    # F  : spreads
        else:         model[iz, ix] += data[t, y]       # Fᵀ : collapses
```

Same loop. Same geometry. Opposite direction of the copy operation.

This is what **"migration is the adjoint of forward modeling"** means concretely.

The velocity–image duality

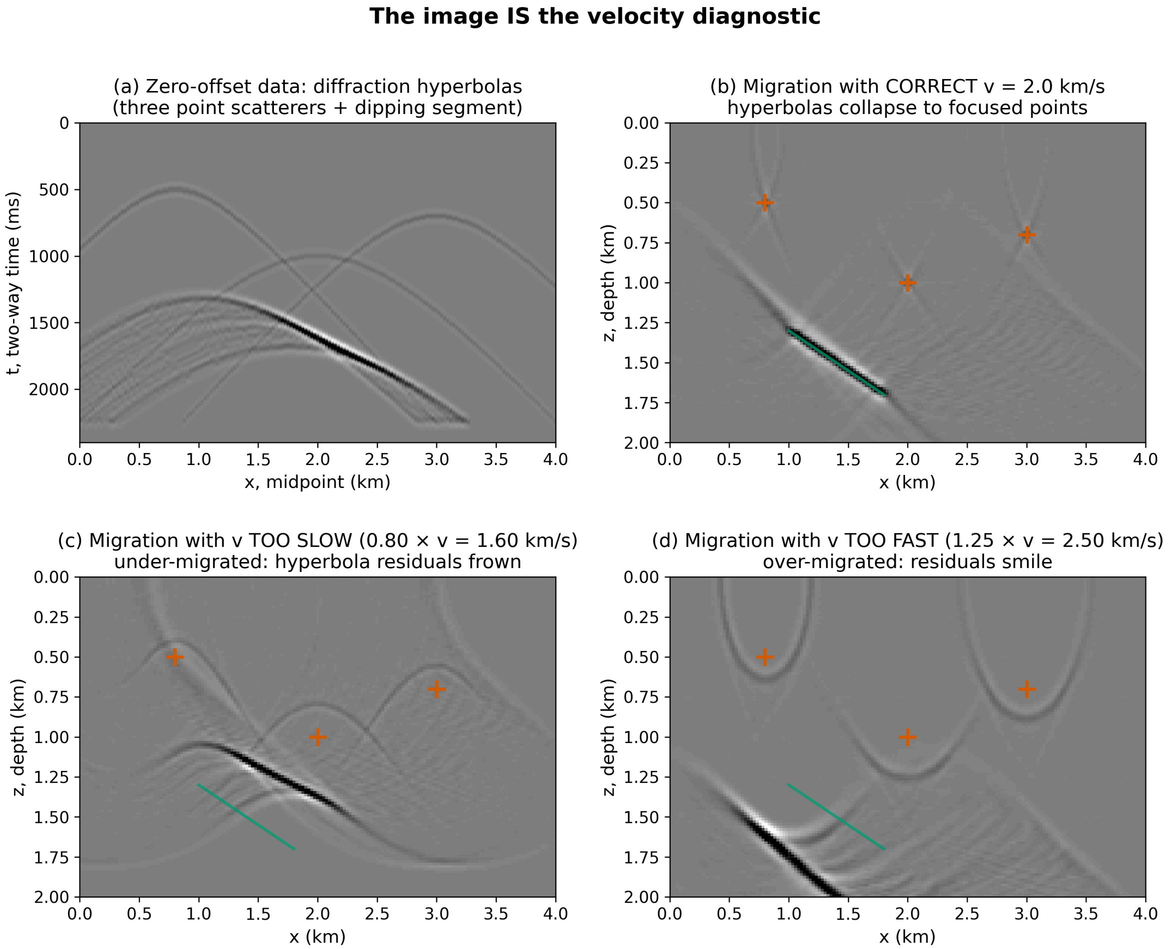
$$m(x,z) = \sum_y w \cdot d\left(y,\ \sqrt{\left(\frac{2z}{v}\right)^2 + \left(\frac{2(x-y)}{v}\right)^2}\right)$$

The summation hyperbola depends on v. Wrong v → wrong hyperbola → residual energy.

Migration velocity	Image signature	Action
Correct	Diffractions collapse to points; flat gathers	Done
Too slow	Downward arcs — **frowns**	Increase v
Too fast	Upward arcs — **smiles**	Decrease v

The image is the velocity diagnostic.

Frowns and smiles



Frowns → migration hyperbola too narrow → velocity too slow.

Smiles → migration hyperbola too wide → velocity too fast.

No borehole needed to read this diagnostic.

Why each method needs the other

	Refraction	Reflection
Measures	Absolute $v_1, v_2, \ldots$	Stacking velocity $V_{\text{rms}}(t_0)$
Depth range	Surface to deepest refractor	Any depth
Strength	Absolute velocity, robust	Full structural image
Blind spot	No LVZ; max refractor depth limited	Relative velocities; near-surface errors compound through Dix

Refraction anchors the velocity. Reflection reveals the structure. Migration fuses both.

The 8-step iterative workflow

Step	Action	Method	Product
1	Pick first breaks	Refraction	$t_{\mathrm{fb}}(x)$
2	Invert first arrivals	Refraction tomography	Shallow $v(x, z)$
3	Pick NMO velocities	Reflection semblance	$V_{\mathrm{rms}}(t_0)$
4	Dix inversion	Reflection	Interval v_{int}
5	Stitch models	Both	Initial $v_0(x, z)$
6	Migrate stacked section	Kirchhoff / RTM	Image $F^\top \mathbf{d}$
7	Diagnose image	Residual moveout	Frowns / smiles / flat?
8	Update v, repeat	Velocity model building	Improved v_1 → Step 6

Loop **6** → **7** → **8** until gathers are flat and diffractions focused.

The feedback that makes iteration work

```
Refraction → v_shallow (absolute)
     ↓
Dix → v_deep (relative, anchored)
     ↓
Migration (Step 6) → image
     ↓
Frowns? → increase v    ←───────┐
Smiles? → decrease v    ←───────┤
Flat?   → DONE          ←───────┘
              ↑
       Velocity update (Step 8)
```

The velocity–image duality converts image quality into velocity corrections — **without any external reference**.

Deep learning as an accelerator

Workflow step	DL application	What it replaces
Step 1	First-break picking U-Net {cite:p}`Mardan2024`	Human picking from intercept-time physics
Between 1–2	Self-supervised denoising {cite:p}`LiTradLiu2024`	Wave-equation signal/noise separation
Steps 2–5	Velocity model building {cite:p}`YangMa2019`	Tomography + NMO + Dix in one pass

Training data for all three networks was produced by **physics-based operators**.

DL **accelerates** the chain. It does not replace the physics upstream of its training data.

Ask this of any surrogate

1. **What physics-based operator** does this network replace?
2. **Who produced the training data**, and what physical knowledge was required?
3. **What is the training distribution** — would you trust this network outside it?

Same questions apply to regression formulas, empirical curves, and neural networks.

The depth of the physics required to answer them is the depth of this course.

Concept check

A zero-offset section shows a diffraction hyperbola with apex at $(x = 3.0\ \mathrm{km},\ t_0 = 1.2\ \mathrm{s})$.

After migration with $v = 2.0$ km/s → **crisp point image**.
After migration with $v = 2.5$ km/s → **upward-curving arc**.

1. Which velocity is more correct, and how can you tell?
2. What depth does the correct image imply?
3. What would $v = 1.5$ km/s produce?

Discuss with your neighbor. Write one sentence on your index card.

Why Hikurangi matters

- Capable of $M_w > 8.5$ megathrust + trans-Pacific tsunami
- Plate interface depth and coupling → building codes, evacuation zones, shakemaps
- Northern margin: unusually **shallow interface** (1–2 km below seafloor near trench) → highest tsunami hazard → only known from seismic imaging

Published campaigns (Wallace et al. 2009; Barker et al. 2018) applied exactly the 8-step workflow: OBS first arrivals → shallow v → reflection migration → focused plate-interface image.

GNS Science programme: https://www.gns.cri.nz/research-projects/hikurangi-subduction-margin/

Lab 4 — Design → Simulate → Image

The lab notebook provides:

- A multi-layer synthetic **forward model** (wave equation simulation)
- A Kirchhoff **migration** routine (~30 lines of NumPy)
- A set of three migration velocity options

Students will:

1. Run the forward model to generate a synthetic zero-offset section
2. Migrate with correct v, $0.80 \times v$, and $1.25 \times v$
3. Report which image is correct and **how they could tell from the image alone**
4. Add a 4th migration velocity from refraction-only input — does it improve or degrade the image?

Tying it together

- **Forward model** $\mathbf{d} = F\mathbf{m}$: given Earth, predict data
- **Migration** $\hat{\mathbf{m}} = F^\top \mathbf{d}$: given data, estimate Earth (requires v)
- **Four cases** of increasing complexity all demand accurate $v(x, z)$:
  flat → multi-layer → dipping → diffractions
- **Refraction + reflection + migration** = one iterative loop to estimate v and build the image
- **Velocity–image duality**: the image itself diagnoses whether v is right
- **Deep learning** accelerates steps; does not replace the physics upstream of its training data

Further reading

- Claerbout (2010). *Basic Earth Imaging*, Ch. 3–5. Open: http://sepwww.stanford.edu/sep/prof/bei11.2010.pdf
- Lowrie & Fichtner (2020). *Fundamentals of Geophysics*, Ch. 3 (UW Libraries)
- Zelt & Barton (1998). Refraction tomography. *JGR* 103, 7187
- Mardan & Fabien-Ouellet (2024). First-break picking U-Net. *Near Surface Geophysics*
- Li et al. (2024). Self-supervised denoising. *Geophysics*
- Yang & Ma (2019). Velocity model building. *Geophysical Journal International*